RESEARCH ARTICLE OPEN ACCESS

Comparison between Unknown Input Estimation of a System Using Projection Operator Approach and Generalized Matrix Inverse Method

Ashis De[*], Gourhari Das[**]
[*](Electrical Engineering Department (P.G., Control System), Jadavpur University, Kolkata-32, India)
[**](Professor, Electrical Engineering Department, Jadavpur University, Kolkata-32, India)

ABSTRACT
In this paper a detailed comparison between the estimation results of unknown inputs of a linear time invariant system using projection operator approach and using the method of generalized matrix inverse have been discussed. The full order observer constructed using projection operator approach has been extended and implemented for this purpose.
Keywords**-** Unknown Input, Unknown Input Estimation(UIE), Linear Time Invariant(LTI) System, Projection Operator, Projection Method Observer(PMO), Unknown Input Observer(UIO), Full Order Observer, Missile Autopilot.

I. INTRODUCTION

Many authors have estimated the unknown input present in a system dynamics by designing an unknown input observer in different approach. Alexander Stotsky and Ilya Kolmanovsky design an observer to estimate the unknown input from available state measurements in automotive control application [1]. Talel. Bessaoudi, Karim Khemiri, Faycal. Ben Hmida and Moncef. Gossa in [2] estimate the unknown input and states of a linear discrete time systems using recursive least-square approach. Estimation of states and unknown input of a nonlinear communication system has been addressed in [3]. In [4], Kalyana C. Veluvolu and Soh Yeng Chai design a high gain observer with multiple sliding mode for state and unknown input estimations. In [5], simultaneous estimation of states and unknown input for a class of nonlinear systems has been proposed by Q. P. Ha and H. Trinh. Thierry Floquet and Jean-Pierre Barbot designed a state and unknown input delayed estimator for discrete-time linear systems [6]. Avijit Banerjee and Prof. G. Das in [7] uses the reduced order Das and Ghoshal observer [9] to estimate the unknown input of a linear time invariant system. Ashis De, Avijit Banerjee and Prof. G. Das in [8] estimate the unknown input of an LTI system using full order observer constructed by the method of generalized matrix inverse [10]. Stefen Hui and Stanislaw H. Zak in [11] designed both unknown input full order and reduced order observer using projection operator approach but they did not give any numerical example of unknown input full order observer and also they did not estimate the unknown input of the given system. In this paper the full order unknown input observer constructed in [11] using projection operator approach has been extended and implemented for estimation of unknown input. With proper numerical example comparison of the estimated results with [8] has been discussed.

Notations: In this paper, $\mathbb{R}^n$ is the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all n×m real matrices. I is the identity matrix and $\emptyset$ is the null matrix with appropriate dimensions. The superscripts "T" and "g" represent the transpose of a matrix and Moore-Penrose generalized matrix inverse(or g-inverse) [15], [16] respectively.

II. MATHEMATICAL PRELIMINARIES

Consider a system described by linear equation,

$$Ax=y \quad (1)$$

where matrix $A \in \mathbb{R}^{m\times n}$, known vector $y \in \mathbb{R}^m$ and unknown vector $x \in \mathbb{R}^n$. Eq. (1) is consistent if and only if,

$$AA^g y = y \quad (2)$$

Now, if eq. (1) is consistent then the general solution of eq. (1) is given by

$$x = A^g y + (I - A^g A)r \quad (3)$$

([16] Graybill 1969, pp. 104). Where $r \in \mathbb{R}^n$ is an arbitrary vector.

III. BRIEF CONSTRUCTION METHODS

Consider a class of LTI system described by the following equations:

$\dot{x} = Ax + Bu + Ev$ (4)

$y = Cx$ (5)

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^{m1}$ is the known input vector, $v \in \mathbb{R}^{m2}$ is the unknown input vector and $y \in \mathbb{R}^p$ is the output vector. A, B, C and E are known constant matrices with appropriate dimensions. Assume that the {A, C} pair is observable and the matrix E is of full column rank.

The system dynamics due to known input u only can be represented as

$\dot{x}_u = Ax_u + Bu$ (6)

$y_u = Cx_u$ (7)

Now subtracting eqn. (6) from eqn. (4) and eqn. (7) from eqn. (5) respectively, the state space model of the auxiliary system can be obtained as

$\dot{x}_v = Ax_v + Ev$ (8)

$y_v = Cx_v$ (9)

where $x_v = x - x_u$ is the system state due to unknown input only and $y_v = y - y_u$ is the output response of the system due to unknown input only.

A. *Estimation using PMO:*

The state x_v can be decomposed as

$x_v = (I - MC)x_v + My_v$ (eqn. (4) of [11]) (10)

From eqn. (5) of [11], it can be written as

$\dot{q} = (I - MC)(Aq + AMy_v)$ (11)

The estimated state is given by the equation

$\hat{x}_v = \hat{q} + My_v$ (eqn. (6) of [11]) (12)

If we add an extra term $L(y_v - Cq - CMy_v)$ to the right hand side of eqn. (11), then the convergence rate will increase. Therefore the dynamics of PMO can be expressed as,

$$\dot{\hat{q}} = (I - MC)(A\hat{q} + AMy_v) + L(y_v - C\hat{q} - CMy_v)$$
$$= \{(I - MC)A - LC\}\hat{q} + \{(I - MC)AM + L - LCM\}y_v \quad (13)$$

The state estimation error, $e = x_v - \hat{x}_v$ and the error dynamic equation is given by,

$\dot{e} = \{(I - MC)A - LC\}e$ since, $x_v - \hat{x}_v = e$. (14)

The unknown input v of the system can be estimated by putting $\hat{x}_v$ in place of x_v in eqn. (8),

$\dot{\hat{x}}_v = A\hat{x}_v + Ev$ (15)

The general solution of eqn. (15) gives the estimated unknown input,

$\hat{v} = E^g\left(\dot{\hat{x}}_v - A\hat{x}_v\right) + (I - E^gE)h_0$ (16)

where h_0 is any arbitrary vector. Since E is of full column rank, $E^gE = I$. Then eqn. (16) reduces to

$\hat{v} = E^g\left(\dot{\hat{x}}_v - A\hat{x}_v\right)$ (17)

The existence of unknown input PMO is governed by the following equation :

$E - MCE = 0$ (pp. 432 of [11]) (18)

Matrix M is calculated from the following equation :

$M = E(CE)^g + M_v\{I - (CE).(CE)^g\}$ (19) (pp. 433 of [11])

Where $M_v \in \mathbb{R}^{m2\times p}$ is an arbitrary matrix and the matrix M is also depends on M_v.

B. *Estimation using generalized matix inverse(or g-inverse) :*

Only required equations from [8] are re-written here for comparison purpose. The existence of unknown input full order observer using g-inverse is governed by the following equation :

$(I - C^gC)E - KCE = 0$ (eqn. (23) of [8]) (20)

The dynamic equation of the observer is given by,

$$\dot{\hat{q}} = (A_v - K_vC_v)\hat{q} + \{(I - C^gC)AC^g - KCAC^g + (I - C^gC)A(I - C^gC)K - KCA(I - C^gC)K\}y_v$$

(eqn. (28) of [8]) (21)

where $A_v = (I - C^gC)A(I - C^gC) - (I - C^gC)E.(CE)^gCA(I - C^gC)$ (eqn. (26) of [8]) (22)

and $C_v = \{I - (CE)(CE)^g\}CA(I - C^gC)$ (23) (eqn. (27) of [8])

The estimated states are given by the equation :

$\hat{x}_v = (I - C^gC)\hat{q} + \{C^g + (I - C^gC)K\}y_v$ (22) (eqn. (29) of [8])

The observer gain is given by,

$K = (I - C^gC)E(CE)^g + K_v\{I - (CE)(CE)^g\}$ (23) (eqn. (24) of [8])

IV. NUMERICAL EXAMPLE

Taking the same numerical example of missile autopilot for MATLAB simulation as described in [8], in which

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -86 & 0 & -12 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -32400 & -216 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -45360 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, E = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 1 & 0 \\ 5 & 1 \end{bmatrix}, M_v = \begin{bmatrix} -5 & 7 \\ 2 & 7 \\ 5 & 8 \\ 5 & 9 \end{bmatrix}$$

Initial condition $x_0 = \begin{bmatrix} 20 \\ 0.25 \\ 1.43 \\ 100 \end{bmatrix}$ and the unknown inputs are taken for simulation as, $v_1 = 10e^{-2t}cos10t$ and $v_2 = 50e^{-5t}sin100t$.

The MATLAB simulation results for the estimation of states and unknown inputs using PMO are shown in Fig. 4.1- 4.6 and the results using g-inverse are shown in Fig. 4.7- 4.12, where red firm lines indicate the actual signals with unknown inputs, green firm lines indicate the responses without unknown inputs and black dotted lines indicate the corresponding estimated signals with unknown input preesnt in a system. From the simulation responses it can be seen that the estimated signals track well the high frequency unknown input signal as well as the low frequency unknown input signal i.e. '$\hat{v}_j$' follows 'v_j'.

A. Estimated results using PMO :

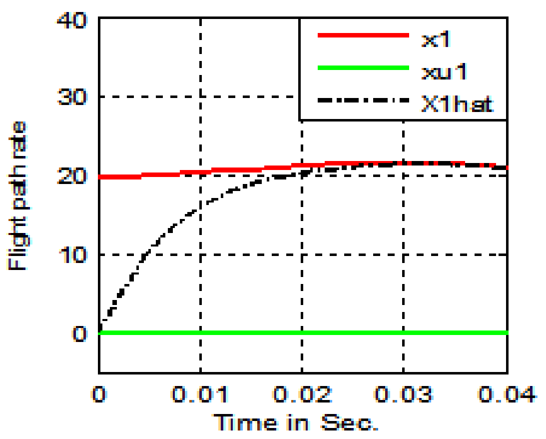

Fig. 4.1: Flight path rate with v_1 & v_2

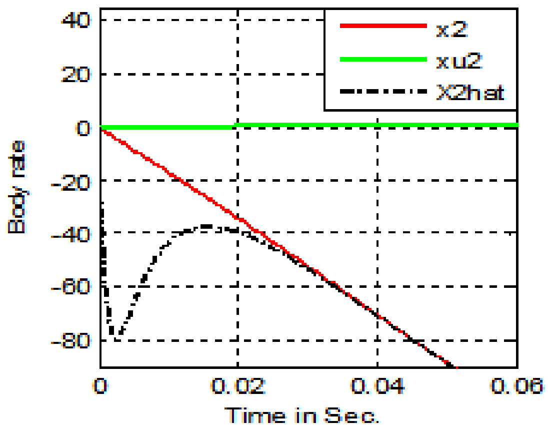

Fig. 4.2: Body rate with v_1 & v_2

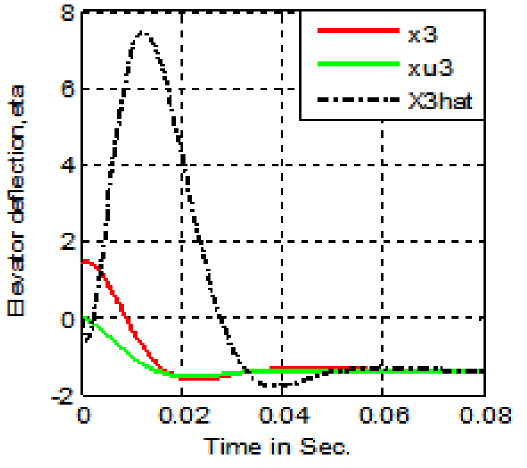

Fig. 4.3: Elevator deflection with v_1 & v_2

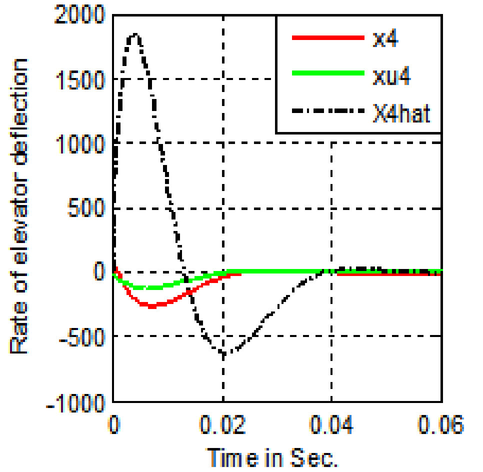

Fig. 4.4: Rate of elevator deflection with v_1 & v_2

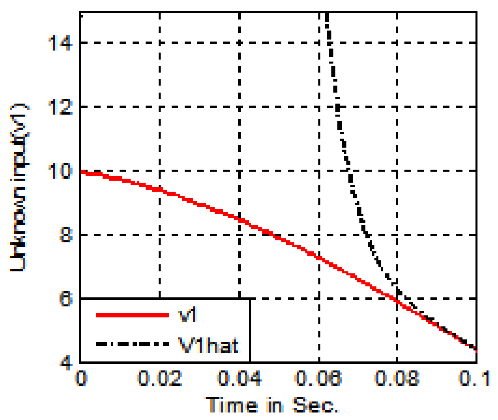

Fig. 4.5: Unknown input(v_1)

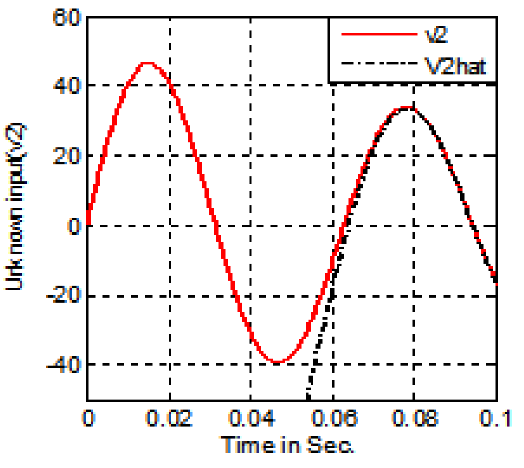

Fig.4.6: Unknown input(v_2)

B. Estimated results using g-inverse :

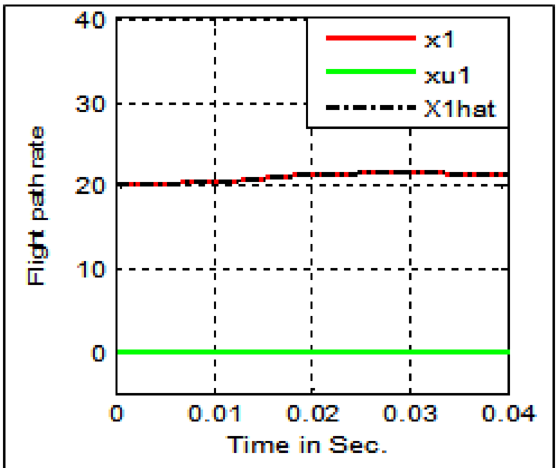

Fig. 4.7: Flight path rate with v_1 & v_2

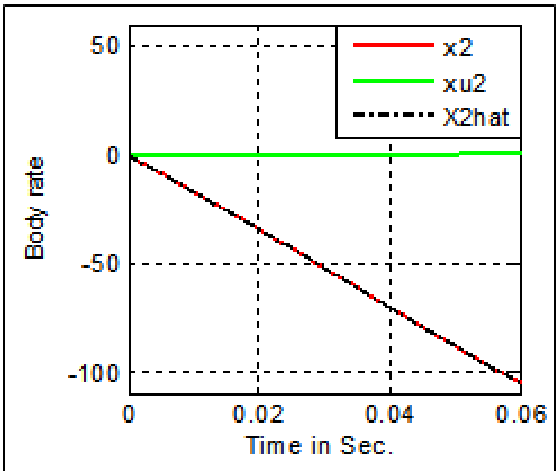

Fig. 4.8: Body rate with v_1 & v_2

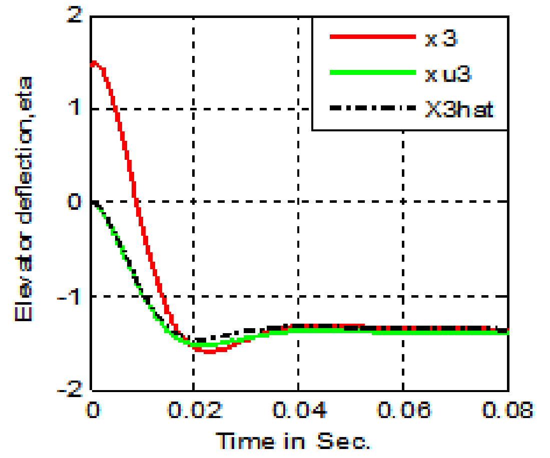

Fig. 4.9: Elevator deflection with v_1 & v_2

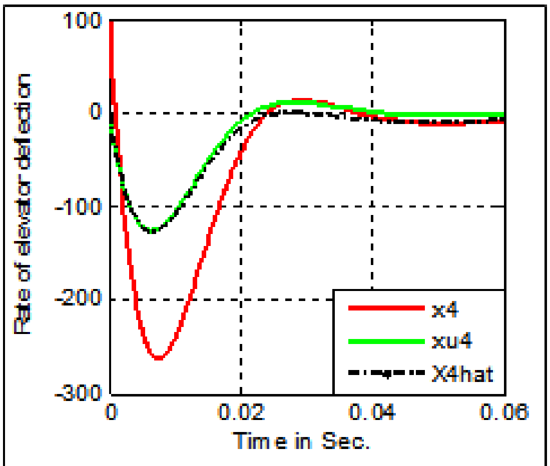

Fig. 4.10: Rate of elevator deflection with v_1 & v_2

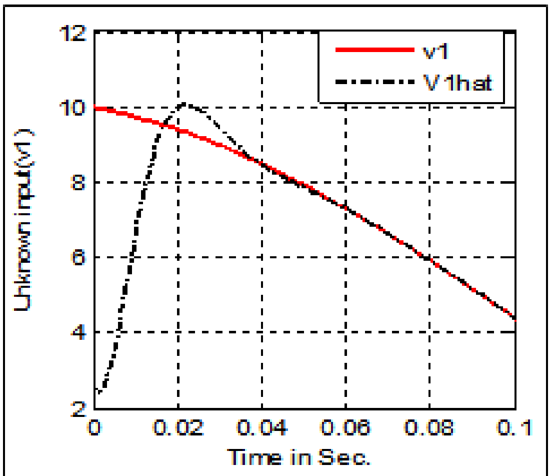

Fig. 4.11: Unknown input(v_1)

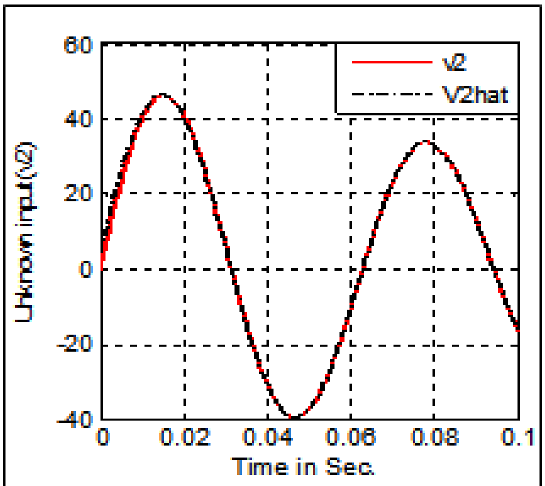

Fig. 4.12: Unknown input (v_2)

V. COMPERATIVE STUDY

A. ***Structured wise comparison (Table- I) :***

Sl. No.	**Unknown input estimation using PMO**	**Unknown input estimation using g-inverse**
1.	The state variable has been decomposed as, $x_v = (I - MC)x_v + My_v$ (eqn. (4) of [11]) where $(I - MC)$ is a projection, not necessarily orthogonal.	The state variable has been decomposed as, $x_v = C^g y_v + (I - C^g C)h$ (eqn. (14) of [8]) where $(I - C^g C)$ is the orthogonal projection of C^g.
2.	The observer dynamic equation in presence of unknown input, $\dot{\hat{q}} = \{(I - MC)A - LC\}\hat{q} + \{(I - MC)AM + L - LCM\}y_v$ {eqn. (13)}	The observer dynamic equation in presence of unknown input, $\dot{\hat{q}} = (A_v - K_v C_v)\hat{q} + \{(I - C^g C)AC^g - KCAC^g + (I - C^g C)A(I - C^g C)K - KCA(I - C^g C)K\}y_v$ (eqn. (28) of [8]) where $A_v = (I - C^g C)A(I - C^g C) - (I - C^g C)E(CE)^g CA.(I - C^g C)$ (eqn. (26) of [8]) and $C_v = \{I - (CE)(CE)^g\}CA(I - C^g C)$ (eqn. (27) of [8])
3.	The observer state variables are given by the equation, $\hat{x}_v = \hat{q} + My_v$ (eqn. (6) of [11])	The observer state variables are given by the equation, $\hat{x}_v = (I - C^g C)\hat{q} + \{C^g + (I - C^g C)K\}y_v$ (eqn. (29) of [8])
4.	The conditions for existence of PMO : 1) $(I - MC)E = 0$ 2) rank(CE) = rank(E).	The conditions for existence of UIO : 1) $(I - C^g C)E - KCE = 0$ (eqn. (23) of [8]) 2) K_v should be chosen such that the eigen values of $(A_v - K_v C_v)$ become negative.
5.	The projection operator $P = I - MC$ is not always idempotant in case of PMO, therefore the Matrix M in eqn. (18) does not have the unique solution, then the observer will be stable for only one value of M.	From the theory of generalized matrix inverse, it is known that the projection ($I - C^g C$) is always symmetric idempotant.

B. ***Performance wise comparison (Table-II) :***

Sl. No.	**Unknown input estimation using PMO**	**Unknown input estimation using g-inverse**
1.	In Fig. 4.1, $\hat{x}_1$ catches x_1 at t = 0.03 sec.	In Fig. 4.7, $\hat{x}_1$ catches x_1 at t = 0 sec.
2.	In Fig. 4.2, $\hat{x}_2$ catches x_2 at t = 0.03sec.	In Fig. 4.8, $\hat{x}_2$ catches x_2 at t = 0 sec.
3.	In Fig. 4.3, $\hat{x}_3$ catches x_3 nearly at t > 0.04 sec.	in Fig. 4.9, $\hat{x}_3$ catches x_3 nearly at t < 0.04 sec.
4.	In Fig. 4.4, $\hat{x}_4$ catches x_4 for t > 0.05 sec.	In Fig. 4.10, $\hat{x}_4$ catches x_4 for t < 0.05 sec.
5.	In Fig. 4.5, $\hat{v}_1$ catches v_1 for t > 0.08 sec.	In Fig. 4.11, $\hat{v}_1$ catches v_1 for t < 0.04 sec.
6.	In Fig. 4.6, $\hat{v}_2$ catches v_2 for t > 0.07 sec.	In Fig. 4.12, $\hat{v}_2$ catches v_2 for t < 0.02 sec.

VI. CONCLUSIONS

In this paper, the projection operator has been used to estimate the unknown inputs by implementing the unknown input projection method observer(PMO). PMO estimates all the four states as well as the unknown inputs. Same numerical example (Missile autopilot) has been taken to estimate the states and unknown inputs as in [8]. Finally after comparision in the discussion section, it can be concluded that the results obtained using generalized matrix inverse is better than the result obtained using PMO.

VII. ACKNOWLEDGEMENTS

Authors would like to thank and acknowledge the teachers of the section for their encouragements and Prabirda, Bimalda, Pankajda, and Saikatda for their extended help in the laboratory.

REFERENCES

[1] Alexander Stotsky, Ilya Kolmanovsky, “Simple unknown input estimation techniques for automotive applications”, *Proceedings of the American Control Conference*, Arlington, VA, pp. 3312-3317, June 2001.

[2] Talel. Bessaoudi, Karim. Khémiri, Fayçal. Ben Hmida and Moncef. Gossa, “Recusive least-squares estimation for the joint input- state estimation of linear

discrete time systems with unknown input" *International Multi- Conference on Systems, Signals & Devices*, 2011

[3] Mohamed Benallouch, Mohamed Boutayeb, Rachid Outbib and Edouard Laroche, "Nonlinear estimation of states and unknown inputs for communication systems", *Proceedings of IEEE International Conference on Signal Processing and Communications (ICSPC 2007)*, Dubai, United Arab Emirates, pp. 696-699, 2007.

[4] Kalyana C. Veluvolu, Soh Yeng Chai. "High gain observers with multiple sliding mode for state and unknown input estimations." *4th IEEE Conference on Industrial Electronics and Applications (ICIEA)*, pp. 1179-1186, 2009.

[5] Q. P. Ha, H. Trinh, "State and input imultaneous estimation for a class of onlinear systems", Automatica 40(2004), pp.1779-1785, May 2004.

[6] Thierry Floquet, Jean-Pierre Barbot, "State and unknown input estimation for linear discrete-time systems", Automatica 42(2006), pp. 1883-1889, Sept. 2006.

[7] Avijit Banerjee, Gourhari Das, "Estimation of unknown input using reduced order Das and Ghoshal Observer", IEEE International Conference on Emerging Trends in Computing, Communication and Nanotechnology (ICECCN 2013), pp. 394-397, 2013.

[8] Ashis De, Avijit Banerjee, Gourhari Das, "Unknown input estimation using full order observer in the domain of generalized matrix inverse", International Journal of Engineering Research and Development, vol. 8, issue 9, pp. 01-05, Sept. 2013.

[9] G. Das and T. K. Ghoshal, "Reduced-order observer construction by generalized matrix inverse", International Journal of Control, vol. 33, no. 2, pp. 371-378, Jan. 1981.

[10] Avijit Banerjee, Partha Pratim Mondal, Gourhari Das, "Construction of Full order observer for linear time invariant systems using generalized matrix inverse", Proceedings of IEEE International Conference on Information and Communication Technologies (ICT 2013), pp. 711-714, 2013.

[11] Stefen Hui, Stanislaw H. Zak, "Observer Design for systems with unknown inputs", Int. J. Appl. Math. Comput. Sci. (amcs), vol. 15, no. 4, pp. 431- 446, 2005.

[12] Avijit Banerjee, Gourhari Das, "Comparison between construction methods of unknown input reduced order observer using projection operator approach and generalized matrix inverse", International conference on Electrical, Electronics Engineering, Bhopal, ISBN: 978-93-82208-47-1, pp. 05-10, Dec. 2012.

[13] Parijat Bhowmick and Prof. Gourhari Das, "Modification of classical two loop autopilot design using PI controller and reduced order observer(DGO)", International Journal of Engineering Research and Development, vol. , issue.7, pp. 01-06, Aug. 2012.

[14] G. Das, K Datta, T. K. Ghoshal, S. K. Goswami, "Structured design methodology of missile autopilot-II", Institute of Engineers (India) Journal, pp. 28-34, 1998.

[15] Victor Lovass-Nagy, Richard J. Miller and David L. Powers, "An introduction to the application of the simplest matrix-generalized inverse in system science", IEEE Transactions on Circuits and Systems, vol. CAS-25, no. 9, pp. 766-771, Sept. 1978.

[16] F.A. Graybill, Introduction to matrices with applications in statistics, Belmont, CA: Wadsworth, 1969.

[17] Katsuhiko Ogata, Modern Control Engineering, PHI learning Pvt. Ltd., 2011.